"SPACE-TIME STRUCTURES" OF SYNERGETICS IN PHYSICAL TERMS OF QUANTUM MECHANICS

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Abstract. Synergetic notion of the space-time structure is defined in physical terms of quantum-mechanical coherent states of the open physical system. Physical interpretation of the process of the space-time structures transformation in time is considered which makes it possible to introduce the notion of time sequence of the space-time structures.

Keywords: synergetics, self-organization, measurement, space-time structures, coherent states, microsystem, macrosystem, living matter

The subject matter of the synergetics — the very advanced branch of scientific knowledge — is the study of origination of the space-time structures in physical systems open for external forces and consisting of identical and possibly "unreliable" elements whose cooperative action constitutes a factor responsible for creation of a space-time structure.

The modern state of this investigation is characterized by the complete lack of definitions of space-time structures in physical terms. This circumstance hampers formulation of the general principles of synergetics as a branch of physics. However, in one of the divisions of the theoretical physics, quantum mechanics — there exists a non-fully analytically worked out problem of description of the quantum-mechanical microsystem-macrosystem interaction in the process of measurement (observation) of physical values, characterizing the quantum-mechanical state of microsystem. Of primary importance for synergetics problems is the fact that the trend of development of quantum measurements theory stated in [1] makes use of the set of notions which are "working" in analysis of specific problems of synergetics discussed in [2]. It would be expedient to stress that investigation into the problem of structures self-organization can be also conducted in terms of quantum measurements theory [1, p. 47–59]. And here, the problem is understood as investigation of a transition of a mechanical open system from the maximum entropy states corresponding to our notion of chaos to the more ordered states whose entropy is lower than that of a thermodynamically equilibrious value. In the author's opinion, such an obvious proximity of the scope of problems in synergetics and in the quantum measurements theory is not accidental and is stipulated by the fact that the space-time structures resulting from cooperative action of a physical system's elements can be adequately described in physical terms of quantum mechanics and, in particular, its subdivision, i.e. the measurements theory.

The given paper suggests an interpretation of synergetic space-time structures in terms of quantum-mechanical coherent states of a microsystem whose states undergo changes in time caused by its interaction with a macrosystem. To make the interpretation analytically comprehensive, below is suggested derivation of the principal evolutionary equations that enable us to give analytical definition of a space-time structure and to introduce the notion of a time sequence of space-time structures. To ensure succession of terminology, we preserve the terms "microsystem" for the measured object and "macrosystem" for the subject of measurement, though such a division is purely conventional in the context of the paper.

Let us begin derivation of the above evolutionary equations with consideration of the evolutionary equation for statistical operator, describing quantum — mechanical state of the system consisting of a microsystem and a macrosystem.

The equation has the form:

$$i\hbar \frac{\partial \rho}{\partial t} = [\mathbf{H}, \rho] \tag{1}$$

And here, H is a system's Hamiltonian operator that can be represented as:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2 + \mathbf{V}$$

where $\mathbf{H}_1$ is a Hamiltonian operator of an isolated microsystem; $\mathbf{H}_2$ is an Hamiltonian operator of an isolated macrosystem; $\mathbf{V}$ is an operator describing a microsystem-macrosystem interaction. All other designations in (1) are generally accepted.

Further, we designate the operations of partial ($\mathrm{Sp}_{1,2}$) tracing of the arbitrary $\boldsymbol{a}$ operator in the following way.

Consider the arbitrary orthonormalized bases consisting of the functions defined over configuration subspace of a microsystem and the functions defined over configuration subspace of a macrosystem. Let it be the set of functions $|k\rangle$ $(k = 1,2,\ldots)$ in the first case, and $|m\rangle$ $(m = 1,2,\ldots)$ in the second.

Here and further on, the use is made of Dirac's designations for the functions and for their complex conjugated values, and for the operators and their matrix elements.

Then operation $\mathrm{Sp}_1\boldsymbol{a}$ is as follows, by definition:

$$\mathrm{Sp}_1\boldsymbol{a} = \sum_{km_1m_2} \langle m_1 k |\boldsymbol{a}| k m_2\rangle \, |m_1\rangle\langle m_2| \,,$$

and operation $\mathrm{Sp}_2\boldsymbol{a}$ by definition is:

$$\mathrm{Sp}_2\boldsymbol{a} = \sum_{mk_1k_2} \langle m k_1 |\boldsymbol{a}| k_2 m\rangle \, |k_1\rangle\langle k_2| \,,$$

Here $|mk\rangle$ denotes the product of the basic functions: $|mk\rangle = |m\rangle\,|k\rangle$ $(m,k = 1,2,\ldots)$, and $|\mathrm{f}\rangle\langle \mathrm{f}|$ is orthogonal projector onto ort $|\mathrm{f}\rangle$.

The partial tracing operations allow to find the statistical operator of a microsystem ρ_1 and the statistical operator of a macrosystem ρ_2 if the statistical operator of the complete system ρ is known. Thus:

$\rho_1 = \mathrm{Sp}_2\,\rho$, $\rho_2 = \mathrm{Sp}_1\,\rho$.

It is easy to notice that $\mathrm{Sp}\rho_i = \mathrm{Sp}\rho$, $(i = 1,2)$. So, ρ_1 and ρ_2 operators represent the statistical operators for a microsystem and a macrosystem in the following sense.

If we consider the arbitrary observable value related to the microsystem and describe it by $\boldsymbol{c}$ operator, then for quantum-mechanical average by the state we obtain $f=\mathrm{Sp}\rho_1\boldsymbol{c}$. Respectively, if we consider the arbitrary observable value related to the macrosystem and describe it by $\boldsymbol{d}$ operator, then we have:

$$b=\mathrm{Sp}\rho_2\boldsymbol{d}.$$

for quantum-mechanical average by condition.

The statistical operator ρ will be further represented as an expansion

$$\rho=\rho_1\rho_2+\boldsymbol{g}, \tag{2}$$

where $\boldsymbol{g}$ is an operator describing the correlation between the observables related to a microsystem and the observables related to a macrosystem. The definition of $\boldsymbol{g}$-operator for the arbitrary observables related to the microsystem and to the macrosystem and described by operators $\boldsymbol{c}$ and $\boldsymbol{d}$, respectively, gives $\mathrm{Sp}\boldsymbol{gc}=\mathrm{Sp}\boldsymbol{gd}=0$. By virtue of arbitrariness of $\boldsymbol{c}$ and $\boldsymbol{d}$ operators it follows from the two above-stated equalities that:

$$\mathrm{Sp}_1\boldsymbol{g} \text{ and } \mathrm{Sp}_2\boldsymbol{g} = 0. \tag{3}$$

Applying successively Sp_2 and Sp_1 operators to equation (1) we obtain evolutionary equations for ρ_1 and ρ_2 statistical operators. This results in the equations:

$$i\hbar\frac{\partial\rho_1}{\partial t} = [\mathbf{H}_1,\rho_1] + [\nu,\rho_1] + \mathrm{Sp}_2[\mathbf{V},\mathbf{g}] \tag{4}$$

$$i\hbar\frac{\partial\rho_2}{\partial t} = [\mathbf{H}_2,\rho_2] + [\omega,\rho_2] + \mathrm{Sp}_1[\mathbf{V},\mathbf{g}] \tag{5}$$

And here $\nu = \mathrm{Sp}_2\mathbf{V}\rho_2$ and $\omega = \mathrm{Sp}_1\mathbf{V}\rho_1$.

While obtaining the above equations, it was taken into account that $\mathrm{Sp}_1[\mathbf{H}_2,\boldsymbol{g}] = 0$ and $\mathrm{Sp}_2[\mathbf{H}_1,\boldsymbol{g}] = 0$, as a consequence of equations (3).

Evolutionary equation for $\boldsymbol{g}$ operator can be obtained from equation (1), substituting into it the expansion (2) and using Eq. (4) and (5) for ρ_1 and ρ_2 statistical operators. As a result, the following equation will be obtained:

$$i\hbar\frac{\partial g}{\partial t} = [\mathbf{H},\boldsymbol{g}] - \rho_1\mathrm{Sp}_1[\mathbf{V},\boldsymbol{g}] - \rho_2\mathrm{Sp}_2[\mathbf{V},\boldsymbol{g}] + [\mathbf{V},\rho_1\rho_2] - \rho_2[\nu,\rho_1] - \rho_1[\omega,\rho_2] \tag{6}$$

The equation is non-homogeneous and linear with respect to $\boldsymbol{g}$ operator.

Suppose that $\mathbf{V}$ operator is small as compared to $\mathbf{H}_1$ and $\mathbf{H}_2$ operators. So, as can be easily seen from the appearance of the Eq. (6), its solution with respect to $\boldsymbol{g}$ operator with initial condition $\boldsymbol{g}$ $(t=t_0) = 0$ (t_0 is the initial moment of time) will have the first order of stipulated smallness since just this order of smallness has got non-homogeneous member in equation (6).

Since the last summands in the right side of equations (4) and (5) have the second order of smallness, we can neglect them.

Due to this, we obtain a system of equations with respect to ρ_1 and ρ_2 statistical operators.

$$i\hbar \frac{\partial \rho_1}{\partial t} = [\mathbf{H}_1 + \nu, \rho_1] \tag{7}$$

$$i\hbar \frac{\partial \rho_2}{\partial t} = [\mathbf{H}_2 + \omega, \rho_2] \tag{8}$$

Now let us express in analytical way the conditions of a system's macroscopicty described by ρ_2 statistical operator. We believe that a macrosystem's description is reduced to a set of the mean values $a_k(t)$ $(k = 1,2,\ldots)$ of the observed macrosystem's physical values to which there correspond a_k operators, i.e. the deviations from the mean values for the observed ones and their correlations can be neglected. Then ν operator should be considered as an operator function of the mean values $\nu = \nu(a_1, a_2, \ldots)$. Since the mean values of these observable ones are varying with time, we'll obtain evolutionary equations for the set $a_k(t)$ $(k=1,2,\ldots)$ submultiplying equation (8) in the right side by the operators a_k $(k=1,2,\ldots)$ and carrying out the tracing operation. Taking account of the mean value definition $a_k = \mathrm{Sp}(\rho_2 \mathbf{A}_k)$ $(k=1,2,\ldots)$, we'll finally obtain the equation of the type:

$$\frac{da_k}{dt} = F_k(a_1, a_2, \ldots \{\rho_1\}),\ k = 1,2,\ldots \tag{9}$$

where F_k is the function of the mean values and the functional of ρ_1 statistical operator. F_k is obtained by averaging, according to a macrosystem's quantum-mechanical state, of G_k operator function of $A_1, A_2, \ldots$ operators and the functional of ρ_1 statistical operators:

$$G_k(A_1, A_2, \ldots \{\rho_1\}) = \frac{i}{\hbar}[H_2 + \omega, A_k] \text{ and}$$

$$F_k(a_1, a_2, \ldots \{\rho_1\}) = \mathrm{Sp}\rho_2 G_k,$$

if deviations from the mean values and correlations of physical values described by $A_1, A_2, \ldots$ — operators are neglected.

The system of equations (7), (9) admits of a solution for a statistical operator ρ_1 in the form of orthogonal projector: $\rho_1 = |\psi\rangle\langle\psi|$. Using the standard designation of a wave function ψ for a state vector $|\psi\rangle$, we write down the principal evolutionary equations allowing an analytical definition of the notion of space-time structures in the open physical systems and their time sequence.

$$i\hbar \frac{\partial \psi}{\partial t} = \boldsymbol{H}(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots)\psi \tag{10}$$

$$\frac{\mathrm{d}\boldsymbol{a}_k}{\mathrm{d}t} = F_k(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots \{\psi\}),\ k = 1,2,\ldots$$

And here $\boldsymbol{H}(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots) = \boldsymbol{H}_1 + \nu(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots)$

Before we proceed to defining the notion of "space-time structure", let us discuss how to interpret this notion within the framework of analytical approach used systematically in [2] and named as "a governing principle".

This notion as well as a physical notion of the "process of space-time structure formation" can be interpreted in terms of mechanical states attained by an open physical system when it is exposed to outward influence. The process of transition from the initial in time mechanical state, which becomes unstable as a result of external action, to the final stable one, can be interpreted as a process of a space-time structure origin. Under space-time structure it is implied the attained final stable mechanical state of an open physical system. In the author opinion, the requirement of instability of the initial mechanical state of an open physical system is restrictive for theoretical self-organization problems. So we suggest a definition of the space-time structures in terms of quantum-mechanical coherent states of an open physical system.

The coherent quantum-mechanical states of an open physical system will be described by wave function $\psi(t)$ satisfying the evolutionary equation (10). Open character of this system will be understood in the following way: the system is subjected to influences on the part of a macrosystem whose mechanical state is characterized by a set of the mean physical values $A_k(t)$ $(k = 1,2,\ldots)$ satisfying the evolutionary equations system (11). Quantum-mechanical process of the space-time structure origin can be understood as follows. In the infinitely far past, a microsystem and a macrosystem being in certain states, began to interact, and in the infinitely far future, as related to the infinitely far past, the process of their interaction leads to formation of a microsystem's quantum-mechanical coherent state which is described by time asymptotics of wave function $\psi(t \to \infty)$. We believe, by definition, that just the asymptotic quantum-mechanical coherent state of an open quantum-mechanical physical system (a microsystem) represents a space-time structure. In the real-state physical systems this process proceeds during the finite time interval. The given circumstance makes it possible to consider in more detail the process of time dynamics in the originated space-time structure.

Such a transformation process will be described as follows.

Let the moment of spatial-temporal structure formation be formally related to the infinitely far past. Then, the space-time structure having arisen from the previous one will be called as asymptotic quantum-mechanical coherent state described by time asymptotics of wave function $\psi(t \to \infty)$. And here, we believe that the previous space-time structure is an asymptotic quantum-mechanical coherent state described by time asymptotics of wave function $\psi(t \to -\infty)$.

The process of space-time structures transformation in the real physical systems also proceeds within the finite time interval similar to the case of the structures origin. This circumstance permits us to repeat many times the aforesaid considerations describing the process of space-time structures transformation (i.e. to date back the moment of time of newly formed space-time structure formally to the infinitely far past, etc.) and to introduce in quite a natural way the notion of time sequence of the space-time structures that we interpreted in terms of quantum-mechanical coherent states of an open physical system.

On introducing the discussed notions we essentially made use of such a mathematical category as time asymptotics of wave function $\psi(t \to \pm\infty)$. Though physical realization of the asymptotic quantum-mechanical coherent states implies that the set of all possible states should be described by the spectrum of such a physical value as energy. This physical requirement can be mathematically expressed as follows: evolutionary operator $\boldsymbol{H}(\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots)$, being included in the equation for wave function (10) which also depends on time $\boldsymbol{H} = \boldsymbol{H}(t)$ through the mean values of $\boldsymbol{a}_i(t)$ $(i = 1,2,\ldots)$, has time $\boldsymbol{H}_{\pm} = \boldsymbol{H}(t \to \pm\infty)$. The eigen values of operators $\boldsymbol{H}_+$ and $\boldsymbol{H}_-$ will be interpreted as energy spectrum of possible quantum-mechanical coherent states whose superposition describes the previous and subsequent space-time structures in time sequence of the space-time structures. Analytical form of this superposition is as follows:

$$\sum_S C_S \exp\left(\frac{\varepsilon_S t}{i\hbar}\right) \varphi_s$$

where $\boldsymbol{C}_S$ is the constants; ε_S is enegry spectrum, φ_S is eigen functions of the operators $\boldsymbol{H}_+$ or $\boldsymbol{H}_-$, $(s = 1,2,\ldots)$.

From the aforesaid analytical representation of quantum-mechanical coherent state termed by us as the space-time structure in the open physical system, it follows that the words "space-time" have sense, since the time dependence in wave function asymtotics will reflect the regulations (structure) of the state's energy spectrum and the laws of the spatial order (structure) are defined by eigenfunctions φ_S.

In conclusion, let us underline methodologic importance of the introduced notions for the problem of the living matter self-organization. In the majority of mathematical models describing the origin of the space-time structures in biological systems there is no comprehensive fundamental physical principles for description of these space-time structures and their dynamics. In other words, in the approaches developed by such sciences as biology, biophysics or biochemistry there is no place for difference between the alive and unalive. Following E. Schrödinger, the authors believe that methodological principle for description of multiform differential stability of the alive should be represented by the quantum theory principle. In methodological aspect, the given paper can be considered as development of this point of view (see for example [3]) within the framework of physics of the alive [4,5].

"ПРОСТОРОВО-ЧАСОВІ СТРУКТУРИ" СИНЕРГЕТИКИ У ФІЗИЧНИХ ТЕРМІНАХ КВАНТОВОЇ МЕХАНІКИ

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У фізичних термінах квантовомеханічних когерентних станів відкритої фізичної системи подається визначенння синергетичного поняття просторово-часової структури. Розглянуто фізичну інтерпретацію процесу перетворення просторово-часової структури з бігом часу, що дозволяє ввести поняття часової послідовності просторово-часових структур.

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В физических терминах квантовомеханических когерентных состояний открытой физической системы дано определение синергетического понятия пространственно-временной структуры. Рассмотрена физическая интерпретация процесса преобразования пространственно-временных структур с течением времени, что позволяет ввести понятие временной последовательности пространственно-временных структур.

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